Gravitational Wave Signatures of Superdense Objects in a Nonsingular Solution of Gravity Theory

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Abstract

A massive gravitationally bound object with a radius $r \leq 2GM/c^2$, which occurs in the non-singular version of the nonsymmetric gravitational theory (NGT), replaces the black hole in Einstein gravity theory. This object is kept stable by the attractive and repulsive forces generated by NGT, as well as standard matter pressures, and is called a superdense object (SDO). The luminosity of gravitational waves emitted by a SDO with a red-shift of order unity is calculated and it is found that it could be a strong source of gravitational radiation at low frequencies. The active galactic nucleus of M87 is identified with a SDO and the recent observational results obtained by the Hubble Space Telesope are used to estimate the amplitude of gravitational radiation.

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Recently, a class of static spherically symmetric solutions of the nonsymmetric gravitational theory (NGT) (Moffat 1979) has been studied and found to contain no event horizons or singularities at $r = 0$ (Cornish and Moffat 1994). A new kind of massive stable, astrophysical object is found to exist, which replaces the black hole in Einstein gravity theory (EGT). This object is kept in hyrodynamic stability by virtue of the attractive gravitational force, and the repulsive NGT force which supplements the standard matter pressures. We call this object a "superdense object" (SDO). Newtonian stars, white dwarfs and neutron stars exist as stable solutions whose hydrodynamic stability is maintained by means of standard matter equations of state (Cornish 1994). Since the SDO can be a stable object even for arbitrarily large masses, it can replace the black hole as a description of very massive and dense astrophysical systems.

In the following, we shall study SDOs as a strong source of gravitational waves, and propose that this phenomenon can be used to observationally distinguish a SDO from a black hole.

The non-singular static spherically symmetric exterior solution of a SDO is characterized by the four parameters M, Q, ℓ^2 and s, denoting the mass, electric charge, NGT charge and a dimensionless parameter, respectively. All the curvature invariants are finite, and the solution for strong fields depends on s in a non-analytic way i.e., there is no smooth limit to EGT for strong fields. This is an important property of non-singular NGT, since for even an infinitesimally small value of s there are no null surfaces in the spacetime and time-like Killing vectors remain time-like everywhere. Thus, there are no black holes in non-singular NGT. The parameter s is expected to be a universal constant.

We shall consider for simplicity SDOs with $Q = 0$ and $\ell^2 = 0$ i.e. electrically and NGT charge neutral stellar objects. The line element is given in spherical polar coordinates

by

$$ds^2=\gamma dt^2c^2-\alpha dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2). \qquad (1)$$

The static spherically symmetric solutions for γ and α are given as expansions in powers of m/r and r/m for large r and for r near zero, respectively, with $0<s<1$ ($m=GM/c^2$). Solutions for $s>1$ can be obtained numerically. For $sm^2/r^2<<1$, the solution is well approximated by the Schwarzschild solution. From this we deduce that the non-singular solution agrees with the standard experimental tests of EGT for a suitably chosen value of the constant s (Cornish and Moffat 1994).

The red-shift between $r=0$ and $r=\infty$ is given for $s<1$ by

$$z=\exp\left[\frac{\pi+2s+\pi/8s^2+\mathcal{O}(s^3)}{2|s|}\right]-1, \qquad (2)$$

while for larger values of s, we have

$$z=\exp\left[\frac{\pi}{\sqrt{2|s|}}\left(1-\frac{\pi|s|-s(1+2\exp(-\pi))+\mathcal{O}(s^0)}{2\pi s^2}\right)\right]-1. \qquad (3)$$

A calculation of the stability of neutron stars for $\ell^2=0$ yielded the bound $s\leq 15$ (Cornish 1994). Thus, for physically allowed values, $s\sim 1-7$, a SDO will be a luminous stellar object with very interesting astrophysical properties. As $s\to 0$ the red-shift will increase rapidly and the SDO will become a dark astrophysical object. We shall assume in the following that s is in the range $1-7$.

Recently, Hubble space telescope (HST) spectra have been obtained giving kinematic evidence for a rotating disk of material in the nucleus of M87 with the radial velocities of the ionized gas measured to be approximately ± 550 km/sec. This yields a mass for the central object $\approx 3\times 10^9\,M_\odot$ and is taken to be strong but preliminary evidence for the existence of black holes (Harms *et al.* 1994).

The unambiguous theoretical and experimental signature of a black hole would be the detection of an infinite red-shift event horizon at the Schwarzschild radius $R_S = 2m$. For M87 an estimate of R_S yields:

$$R_S = 8.8 \times 10^{14}\,\mathrm{cm}, \tag{4}$$

which is four orders of magnitude less than the closest point to the center of M87 that can be observed with the HST. Thus, it is not possible at present to obtain direct experimental evidence that the nucleus of M87 contains a black hole. Moreover, the object contained in the nucleus of M87, given its estimated size (> 60 light yrs), could well be a stable system of stars with a lifetime $\sim 10^{11}$ yrs.

But there is the alternative possibility that the object is a stable SDO with a mass $M \sim 3 \times 10^9\, M_\odot$. Most of the mass will be concentrated near the center of the SDO. Such an object can possess very different astrophysical properties from a black hole, which may allow it to be distiguished experimentally from the latter object.

It is possible that SDOs are formed in the early universe and we call these objects primordial SDOs. The other possibility is that they are formed by collapse of a massive star or through accretion of large numbers of stars in a galaxy. A SDO can be identified with supermassive compact objects with densities exceeding those of neutron stars. They can also be identified with large stable, massive objects such as those found in active galactic nuclei.

The gravitational wave energy radiated by a non-spherical self-gravitating system is given by (Misner, Thorn and Wheeler 1973; see also Shapiro and Teukolsky 1983):

$$E_G \sim Mc^2\left(\frac{r_G}{R}\right)^{7/2}, \tag{5}$$

where M and R are the characteristic mass and size of the source, respectively, and $r_G = GM/c^2$. The power output of gravitational waves is of the order:

$$\frac{dE}{dt} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)v^6 \sim \left(\frac{r_G}{R}\right)^2\left(\frac{v}{c}\right)^6 L_0, \tag{6}$$

where

$$L_0 \equiv \frac{c^5}{G} = 3.6 \times 10^{59}\,\mathrm{erg\,sec^{-1}}, \tag{7}$$

and v is the velocity of the source. Eliminating v/c from (5), we get

$$L_{GW} \sim \left(\frac{r_G}{R}\right)^5 L_0. \tag{8}$$

We can parametrize (4) and (7) by writing:

$$\Delta E_G = Mc^2\epsilon^{9/2}\delta^{7/2}, \quad \delta = \frac{r_G}{R}, \quad \epsilon = \frac{\Delta M}{M}, \tag{9}$$

and

$$L_{GW} \sim \delta^5\epsilon^5 L_0, \tag{10}$$

where ϵ is the mass fraction expected to produce the amplitude of gravitational radiation.

A gravitational wave will generate a small relative acceleration on test particles given by

$$\ddot{\xi}_i = \frac{1}{2}\ddot{h}^{TT}_{ik}\xi_k, \tag{11}$$

where h^{TT}_{ik} denotes the spatial components of the traceless–transverse weak field symmetric metric. This produces a small change in the separation of the particle of the size:

$$\delta\xi_i = \frac{1}{2}h^{TT}_{ik}\xi_k. \tag{12}$$

Then, we have

$$h \sim \frac{\delta\xi}{\xi} \sim \delta\left(\frac{r_G}{r}\right)\epsilon^2 \sim 1.5 \times 10^5\left(\frac{M/M_\odot}{r}\right)\epsilon^2. \tag{13}$$

The characteristic time scale for our gravitational wave source is

$$T \sim \left(\frac{R^3}{GM}\right)^{1/2} \epsilon^{-1/2}, \tag{14}$$

and the damping time from gravitational radiation is approximately:

$$\tau_d \sim \frac{R}{c}\left(\frac{R}{r_G}\right)^3 \epsilon^{-3}. \tag{15}$$

Let us assume that the size of the SDO describing the core of M87 is of the order of the Schwarschild radius:

$$R_S \equiv 2r_G = 8.8 \times 10^{14}\,\mathrm{cm}, \tag{16}$$

and that the mass fraction that takes part in the vibration of the SDO is $\epsilon \sim 10^{-4}$. Then, from (8) we find that

$$\Delta E_G \sim 5.4 \times 10^{45}\,\mathrm{erg}. \tag{17}$$

The mean frequency is given by

$$\frac{1}{T} \sim 2.4 \times 10^{-7}\,\mathrm{Hz}. \tag{18}$$

and ignoring damping by turbulence, heat conductance and other effects, the gravitational radiation damping time is

$$\tau_d \sim 2.3 \times 10^{17}\,\mathrm{sec} \sim 6 \times 10^{10}\,\mathrm{periods}. \tag{19}$$

The gravitational wave power output will be of the order:

$$L_{GW} \sim 1.1 \times 10^{38}\,\mathrm{erg/sec}. \tag{20}$$

For the approximate distance of M87, $d = 1.7 \times 10^7$ pc, the flux measured at Earth will be given by

$$F \sim 3.2 \times 10^{-15}\,\mathrm{erg/cm^2/sec}. \tag{21}$$

The fraction of mass of the SDO that produces the gravitational wave amplitude could be much less than the value we have assumed. The efficiency of the gravitational wave output would be greater for the collision of two SDOs.

For $\delta \sim 1$, we obtain from (12) for the displacement of the end of a 10 m bar on the Earth, assuming that $\epsilon \sim 10^{-4}$:

$$\delta\xi \sim 8.7 \times 10^{-17}\,\mathrm{cm}. \tag{22}$$

Weber's original bars were sensitive to about $h \sim 10^{-16}$ (Weber 1960) and new bars will be sensitive to 10^{-20} or better, so if the bar or interferometer is tuned to the low frequency, $\sim 10^{-6} - 10^{-7}$ Hz, then there is a possibility to measure this strong output of gravitational radiation from the core of an active galactic nucleus. Of course, with $\delta \sim (0.1 - 0.01)R_S$ there will be a significant increase in the gravitational wave flux but with a decrease in the time that the source produces radiation.

A gravitational wave detector with the sensitivity of the space-based gravitational wave interferometer LISA/SAGITTARIUS could see gravitational wave events involving SDOs in the frequency range $10^{-6} - 1$ Hz (Danzmann *et al.* 1993, Hellings *et al.* 1993). Earth based detectors would have difficulty seeing events at such low frequencies due to unavoidable seismic noise. The question whether a population of supermassive black holes is likely to emit detactable gravitational waves has been studied in the literature (see Schutz 1992 for a review). In principle, a significant fraction of the rest mass energy could be released during the formation of a black hole, if it involves a non-axisymmetric collapse or rotation phase. But calculations have shown that for stellar mass size sources the emission of gravitational waves would be small (Rees 1984; Haehnelt 1994). Another possibility is for a head-on collision of a super massive black hole with a compact stellar object or the capture of such an object into a relativistic orbit. But none of these possible sources of black

hole gravitational wave emission are expected to produce as strong a signal as the SDO, so that the detection of a strong flux of gravitational waves at a frequency $\sim 10^{-6} - 10^{-7}$ Hz could be interpreted as preliminary evidence for the existence of luminous SDOs.

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